Deconfining phase transition in pure gauge theories

1. Polyakov loop and *hidden* global symmetries

2. Z(3) interface tension in SU(3)

   Potential for constant A_0

3. Effective model for pure glue

4. “Birdtrack” diagrams

5. Gross-Witten-Wadia transitions at large N for SU(N)

Polyakov loop and hidden global symmetries

"Hidden" symmetry

In QCD there is a *local* SU(3) gauge symmetry:

$$A_\mu(x) \to \frac{1}{-ig}\,\Omega^\dagger(x)\left(\partial_\mu - igA_\mu(x)\right)\Omega(x)$$

Gauge transformation $\Omega(x)$ differs at *each* x.

Subset of local: *global* gauge transf's, $\Omega(x) = \Omega$:

$$A_\mu(x) \to \Omega^\dagger A_\mu(x)\,\Omega$$

So what? Ω is a SU(3) matrix, so det $\Omega = 1$. Consider

$$\Omega_1 = \begin{pmatrix} \mathrm{e}^{2\pi i/3} & 0 & 0 \\ 0 & \mathrm{e}^{2\pi i/3} & 0 \\ 0 & 0 & \mathrm{e}^{2\pi i/3} \end{pmatrix} = \mathrm{e}^{2\pi i/3}\,\mathbf{1}$$

det $\Omega_1 = (\mathrm{e}^{2\pi i/3})^3 = 1$:

Ω_1 *is* a SU(3) matrix

But: Ω_1 is ~ to *unit* matrix!

Global Z(3) symmety

Because Ω_1 is proportional to unit matrix, gluons invariant:

$$A_\mu(x) \to \Omega_1^\dagger A_\mu(x)\,\Omega_1 = \mathrm{e}^{-2\pi i/3}\,A_\mu(x)\,\mathrm{e}^{2\pi i/3} = A_\mu(x)$$

Quarks are *not*, since they pick up a phase

$$q(x) \to \Omega(x)\,q(x)\ ;\ \Omega_1\,q = \mathrm{e}^{2\pi i/3} q \neq q$$

There are three such phases: global Z(3) symmetry hidden in SU(3)

$$\Omega_1 = \mathrm{e}^{2\pi i/3}\,\mathbf{1}\ ,\ \Omega_2 = \mathrm{e}^{-2\pi i/3}\,\mathbf{1}\ ,\ \Omega_3 = \mathbf{1}$$

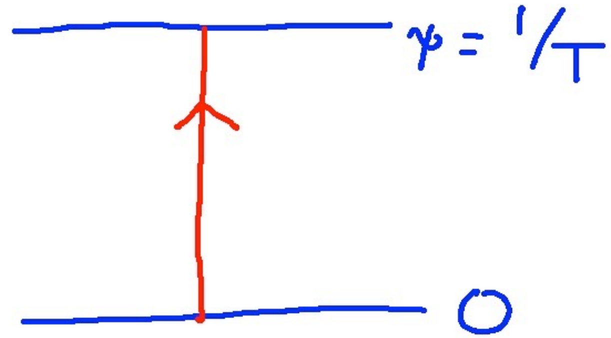
With*out* quarks, *absolute* measure of confinement/deconfinement, ~ Z(3)

With quarks, only *approximate* measure: for QCD, good or bad approximation?

Lines and Loops

Consider Wilson line in (imaginary) time direction:

$$\mathbf{L}(\vec{x}) = \mathcal{P}\exp(ig\int_0^{1/T} A_0(\vec{x},\tau)d\tau)$$



Like propagator of heavy quark. Under a gauge transformation,

$$\mathbf{L}(\vec{x}) \rightarrow \Omega^\dagger(\vec{x}, 1/T)\,\mathbf{L}(\vec{x})\,\Omega(\vec{x},0)$$

If *only* gluons, can choose gauge transf's periodic *only* up to Z(3):

$$\Omega(\vec{x}, 1/T) = \mathrm{e}^{2\pi i/3}\,\Omega(\vec{x},0)$$

Trace of Wilson line = Polyakov-Susskind loop is gauge invariant up to Z(3)

$$\ell(\vec{x}) = \frac{1}{3}\,\mathrm{tr}\,\mathbf{L} \rightarrow \mathrm{e}^{2\pi i/3}\,\ell(\vec{x})$$

Confinement as Z(3) domains

Confining vacuum:

domains of Z(3) phases,

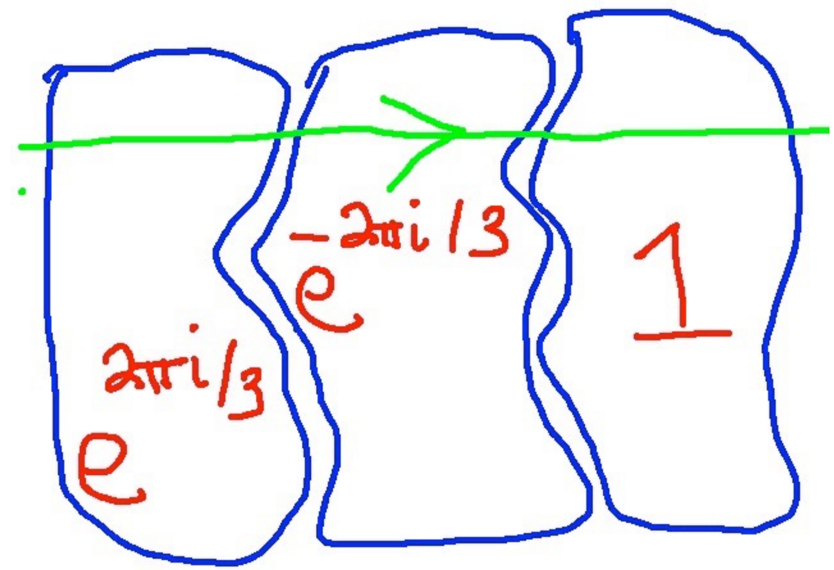
randomly disordered.

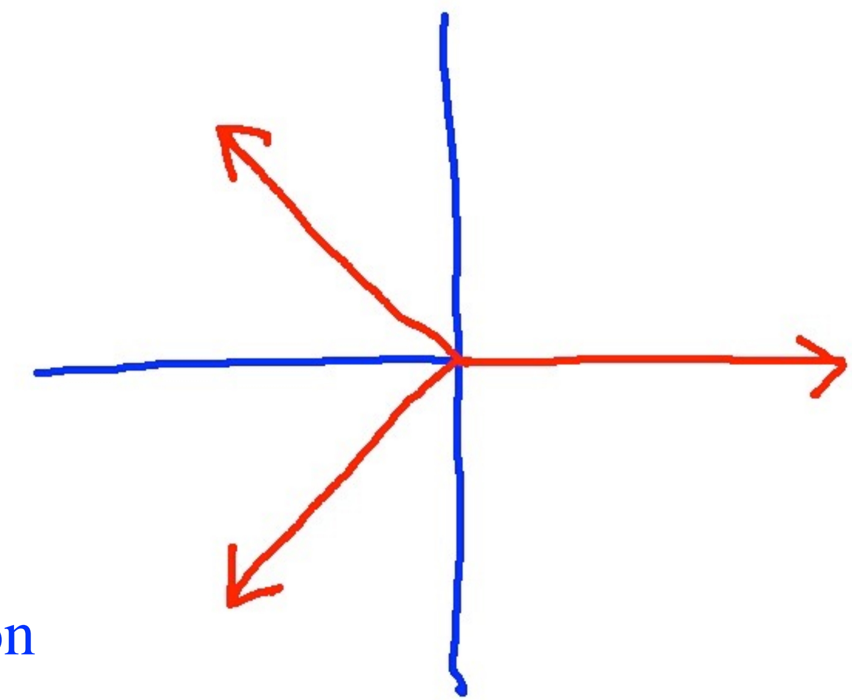
Propagation through domains:

phase is random, averages to zero,

so $\langle propagator \rangle \sim \langle loop \rangle = 0$

$$e^{2\pi i/3} + e^{-2\pi i/3} + 1 = 0$$





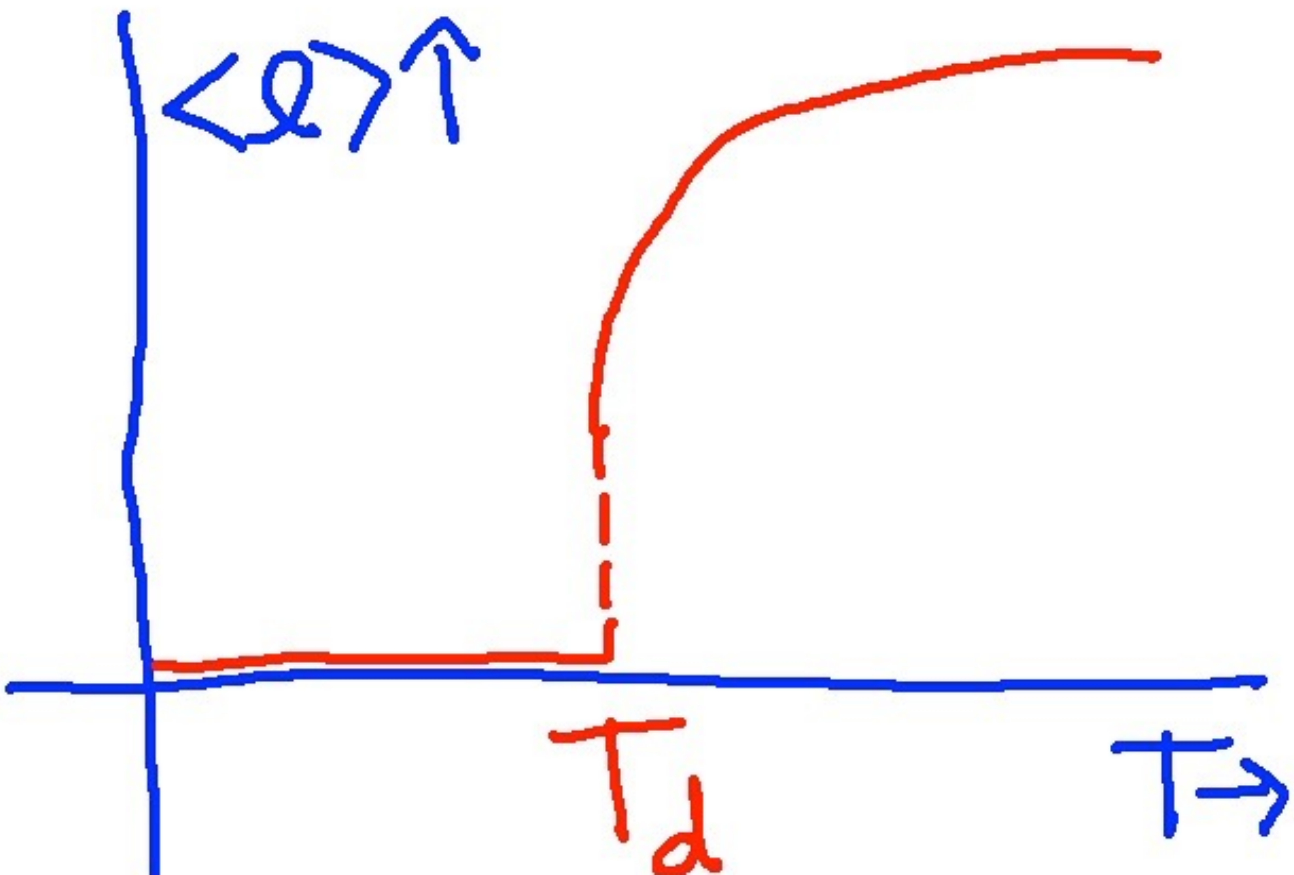
Confinement is not infinite (effective) mass
but phase decoherence, ~ Anderson localization

Deconfinement at temperature $T \neq 0$

As $T \to \infty$, $g^2(T) \sim 1/\log(T)$. Hence A_0 is small, $\langle loop \rangle \sim 1$.

Two phases: *confining*, $T < T_d$: $\langle loop \rangle = 0$. *Deconfining*, $T > T_d$: $\langle loop \rangle > 0$

First order for 3 colors: cubic invariant from Z(3) symmetry.



Z(3) interface tension, potential for A_0.

Z(3) degenerate vacua

Consider a classical field, $$A_0^{cl} = \frac{2\pi T}{3g}\, q\, t_8 \qquad t_8 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

If we take q = 1, 2, or 3, we get Z(3) states

$$\mathbf{L}(A_0^{cl}) = \mathrm{e}^{2\pi i j/3}\, \mathbf{1}$$

But what about *arbitrary* constant q? As constant, diagonal field, $G_{\mu\nu} = 0$.

$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$$

So then *any* q is a vacuum? Can't be right, should only be 3 vacua.

Will show: classical degeneracy lifted by quantum effects.

Compute at high temperature, so semi-classical expansion should be ok.

Gross, RDP, Yaffe '81; Weiss, '82; Bhattacharya, Gocksch, Korthals-Altes & RDP, '91

Lifting the degeneracy

Expand about classical field to one loop order,

$$A_\mu = A^{cl}_\mu + A^{qu}_\mu \; , \; A^{cl}_\mu = \delta_{\mu 0} \, \frac{2\pi T}{3g} \, q \, t_8$$

Use the background field method, L. Abbott, Nucl Phys B185, 181 (1985)

$$\mathcal{S}^{qu} = \frac{1}{2} \, \mathrm{tr} \, \log(-D^2_{cl})$$

This is valid only for the above classical field. We need to evaluate

$$D^{cl}_\mu A^{qu}_\nu = \partial_\mu A^{qu}_\nu - ig[A^{cl}_\mu, A^{qu}_\nu]$$

At T ≠ 0

$$i\partial_0 = p_0 = \; 2\pi T \, n \; , \; \; n = 0, \pm 1, \pm 2 \ldots \qquad \int \frac{dk_0}{2\pi} \to T \sum_n$$

Tricks to compute

How to deal with $[A_0^{cl}, A_\nu^{qu}]$? Useful to us ladder generators, as for SU(2):

$$t_4^+ = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \; ; \; t_4^- = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \mathrm{tr}(t_4^+ t_4^-) = 1 \; , \; \mathrm{tr}(t_4^+ t_4^+) = \mathrm{tr}(t_4^- t_4^-) = 0$$

Very simple commutator! $\quad [t_8, t_4^\pm] = \pm\, 3\, t_4^\pm$

Then *all* of the messy SU(3) matrices collapse, as

$$D_0^{cl} A_\nu^{qu,4\pm} = \partial_0 A_\nu^{qu,4\pm} - ig[A_0^{cl}, A_\nu^{qu,4\pm}] = -i\, 2\, \pi T (n \pm q) A_\nu^{qu,4\pm}$$

In background A_0, *all* p_0's get shifted from $2\pi T$ * integer to

*$2\pi T$ * (integer + q).*

More tricks: sum over "n" last

We need

$$\mathcal{V}^{qu}(q) = 4\,\mathrm{tr}\log((p_0^+)^2 + \vec{p}^{\,2}) \; , \; p_0^+ = 2\pi T(n+q)$$

We assume that "q" is *constant*, and so this is a potential, $V^{\rm qu}$(q).

When in doubt, differentiate

$$\frac{\partial}{\partial q}\mathcal{V}^{qu}(q) = 8\,(2\pi T)\mathrm{tr}\frac{p_0^+}{(p_0^+)^2 + \vec{p}^{\,2}}$$

In the loop, *first* integrate over spatial p!

$$(16\pi T)T\sum_{n=-\infty}^{+\infty}\int\frac{d^3p}{(2\pi)^3}\frac{p_0^+}{(p_0^+)^2+\vec{p}^{\,2}} = -16\pi^2T^4\sum_{n=-\infty}^{+\infty}(n+q)|n+q|$$

ζ functions

Zeta functions are very useful:

$$\zeta(r,q) = \sum_{n=1}^{\infty} \frac{1}{(n+q)^r}$$

Turn the sum over all "n" into positive "n",

$$\sum_{n=-\infty}^{+\infty} (n+q)|n+q| = \zeta(-2,q) - \zeta(-2,1-q)$$

Useful identity:

$$\zeta(-2,q) = -\frac{1}{12}\frac{d}{dq} q^2(1-q)^2$$

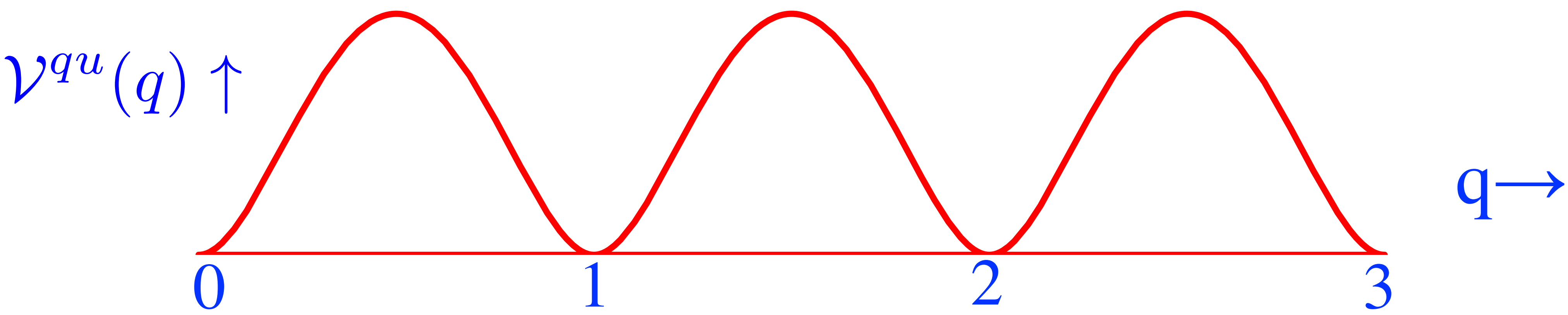
We finally get

$$\mathcal{V}^{qu}(q) = \frac{8\pi T^4}{3}\, q^2(1-q)^2$$

Lifting the degeneracy

Quantum fluctuations generate a potential for q, $\mathcal{V}^{qu}(q) = \dfrac{8\pi T^4}{3}\, q^2(1-q)^2$

$$\mathbf{L}(q) = \mathrm{e}^{2\pi i j/3}\,\mathbf{1} \;\; if \;\; q = j$$



q = 0 and 3 are the same, just shows that q is a periodic variable.

Non-trivial: q = 1 and 2 are degenerate with q = 0 , 3 : Z(3) symmetry!

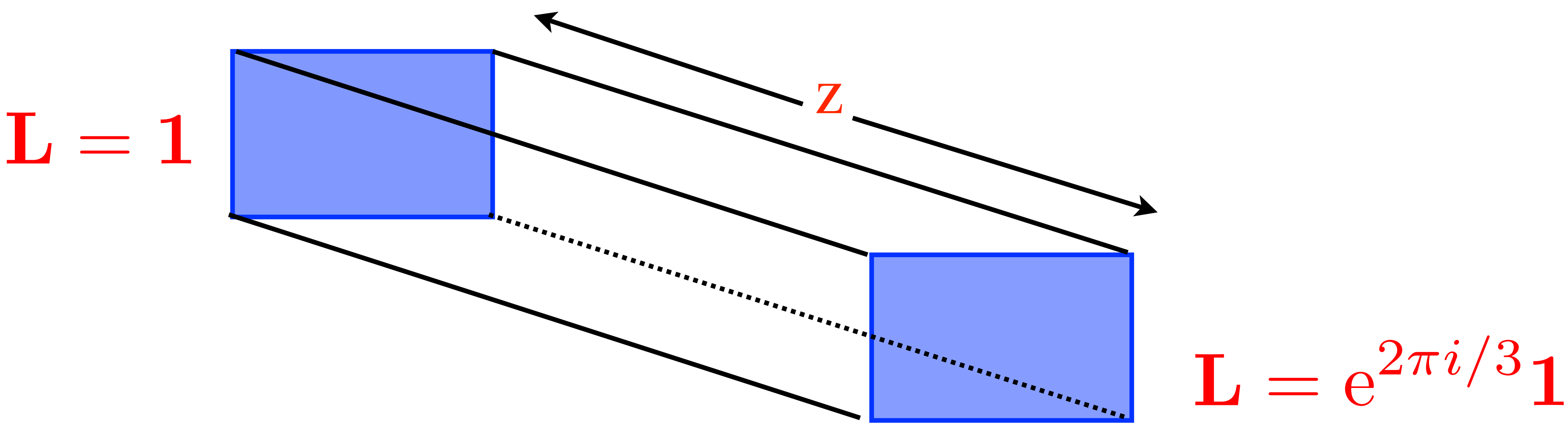
N.B.: above potential is *only* valid for $0 \leq q \leq 1$: periodic, as shown, for other q.

V(0) includes the free energy of massless gluons, $- 8\,\pi^2\, T^4/45$.

So what? Z(3) interface tension

Consider a box which is long in one (spatial) direction, with one vacuum

at one end, and a different, but degenerate vacua, at the other.



Between the two vacua, an *interface* forms, with finite energy $\sim V_{tr}$.

In weak coupling, we can compute this using the potential above.

A tunneling problem

Now let q, which was constant, become q(z).

The classical action is $$\frac{1}{2}\mathrm{tr}(G^{cl}_{\mu\nu})^2 \sim \left(\frac{dA_0^{cl}}{dz}\right)^2 = \frac{8\pi^2 T^2}{3\,g^2}\left(\frac{dq}{dz}\right)^2$$

We can combine the two, to get

$$\mathcal{S}^{cl} + \mathcal{V}^{qu} = V_{tr}\,\frac{8\pi^2 T^3}{3\,g}\int d\widetilde{z}\left(\left(\frac{dq}{d\widetilde{z}}\right)^2 + q^2(1-q)^2\right)\,,\ \widetilde{z} = g\,T\,z$$

The classical term is $1/g^2$; the quantum potential, g^0.

Balancing the two gives an action in between, ~ $1/g$ ($= \sqrt{g^2}$) $$\sigma_{Z(3)} = \frac{8\pi^2}{9}\,\frac{T^3}{\sqrt{g^2}}$$

At small g the interface is wide, ~ $1/gT$: approx constant q ok.

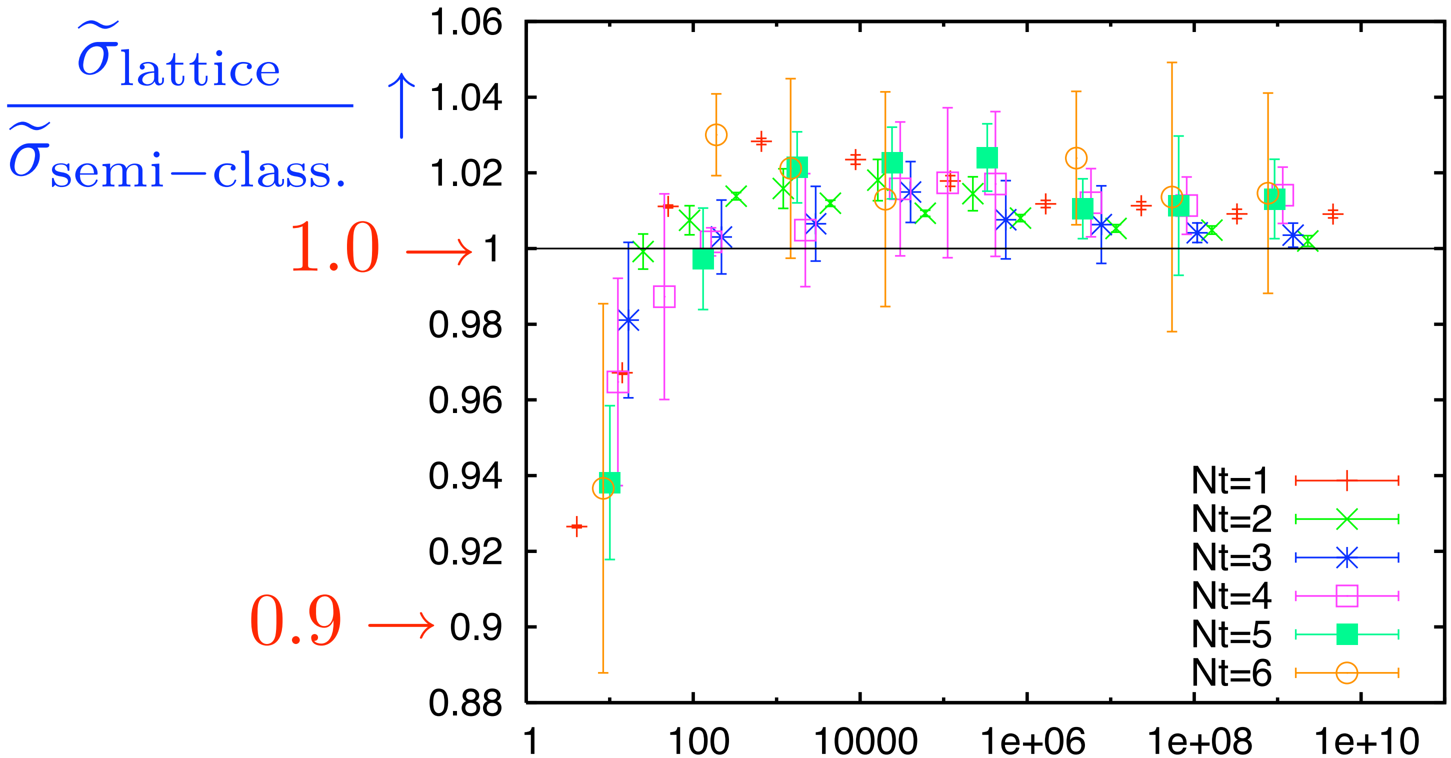
Lattice: Z(N) interfaces = ‘t Hooft loop

From lattice: semi-classical Z(N) interface tension works well to ~ 10 T_c .

$\sigma_{Z(3)}$ ~ ‘t Hooft loop: Korthals-Altes, Kovner & Stephanov, hep-ph/9909516

For N ≥ 4, several interface tensions: satisfy semi-classical relation down to T_d:
Bursa & Teper, hep-lat/0505025

$$\widetilde{\sigma}_k = \frac{k(N-k)}{N-1}\,\widetilde{\sigma}_1$$



←de Forcrand & Noth, hep-lat/0506005.

$$\mathbf{L} : \mathbf{1} \rightarrow e^{2\pi i k/N}\,\mathbf{1}$$

$T/\Lambda_{\overline{MS}}$ →

Results from the lattice, pure glue and not

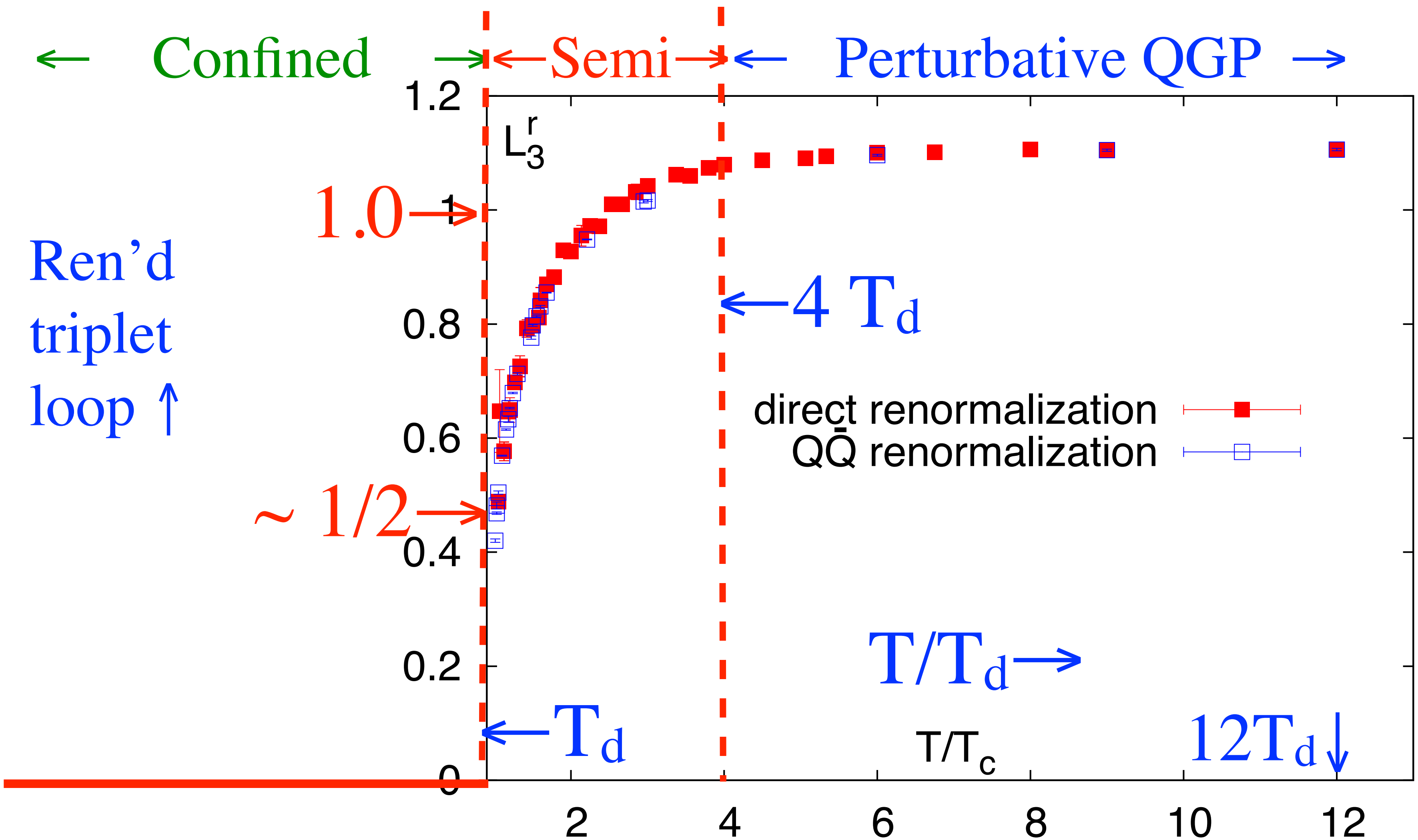
Lattice: renormalized loop, no quarks

Renormalized loop from lattice: Gupta, Hubner & Kaczmarek 0711.2251.

$\langle loop \rangle = 0$, T < T_d. Confined phase

1/2 < $\langle loop \rangle$ < 1, T: T_d. → 4 T_d , "semi" QGP, *partially* deconfined. *Broad* region

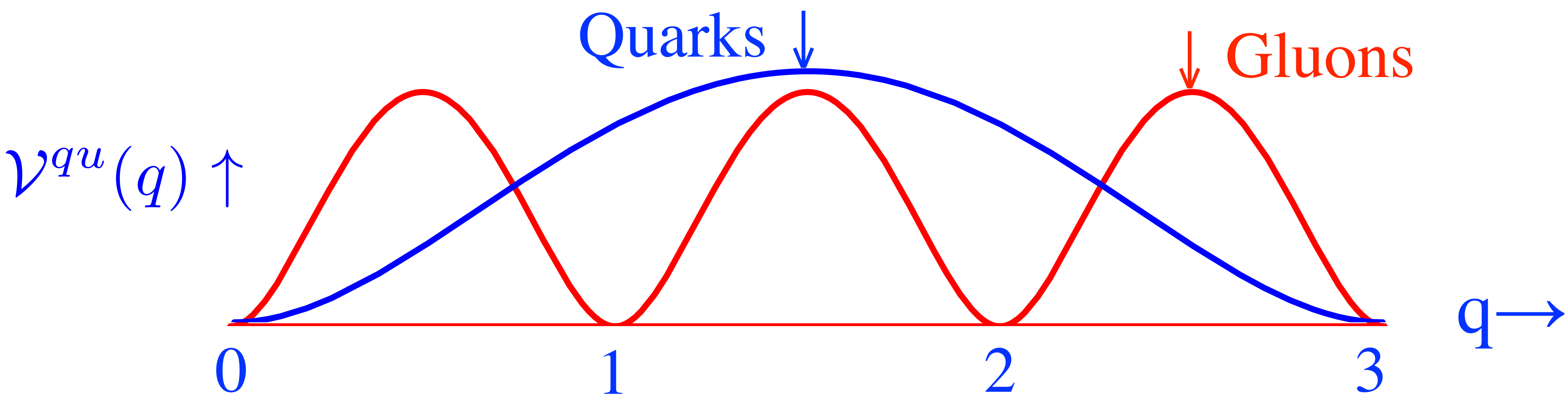
$\langle loop \rangle \sim 1$, T > 4 T_d , perturbative QGP



Potential for A_0, with quarks

Including the potential with quarks, $A_0^{cl} = \frac{2\pi T}{3g}\, q\, t_8$

$\mathbf{L}(q) = \mathrm{e}^{2\pi i j/3}\, \mathbf{1}$ *if* $q = j$



Above for 3 massless flavors.

With quarks, the Z(3) vacua with q = 1 and 2 are no longer degenerate.

Dynamical breaking of Z(3) symmetry by dynamical quarks.

Lattice: renormalized loop, with quarks

With quarks, <*loop*> ≠ 0 at *any* T ≠ 0

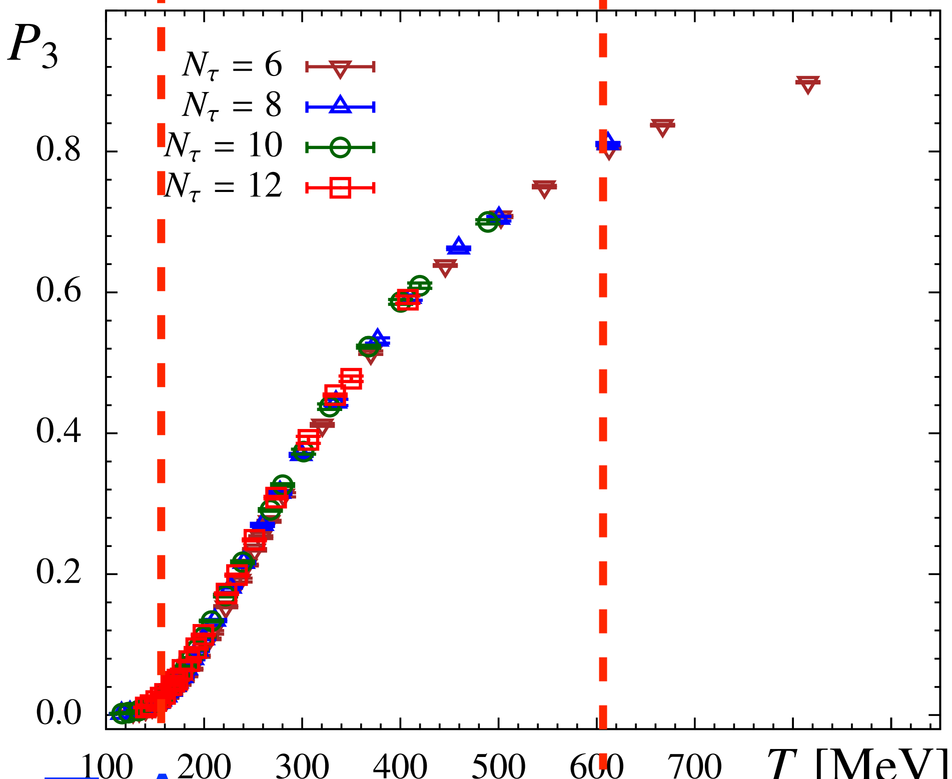
Lattice: QCD, 2+1 flavors. $T_\chi \sim 155$ MeV, *crossover*.

Ren.'d Polaykov loop *very* small at T_χ, semi-QGP until ~ 3 T_χ.

Broad in T: *why does HRG fail at ~ 140 MeV? Why χSB'g in ~ confined phase?*

← Hadronic → ←Semi-QGP→ ←Perturbative QGP

Ren.'d triplet loop ↑



T_χ ↑

Petreczky & Schadler, 1509.07874

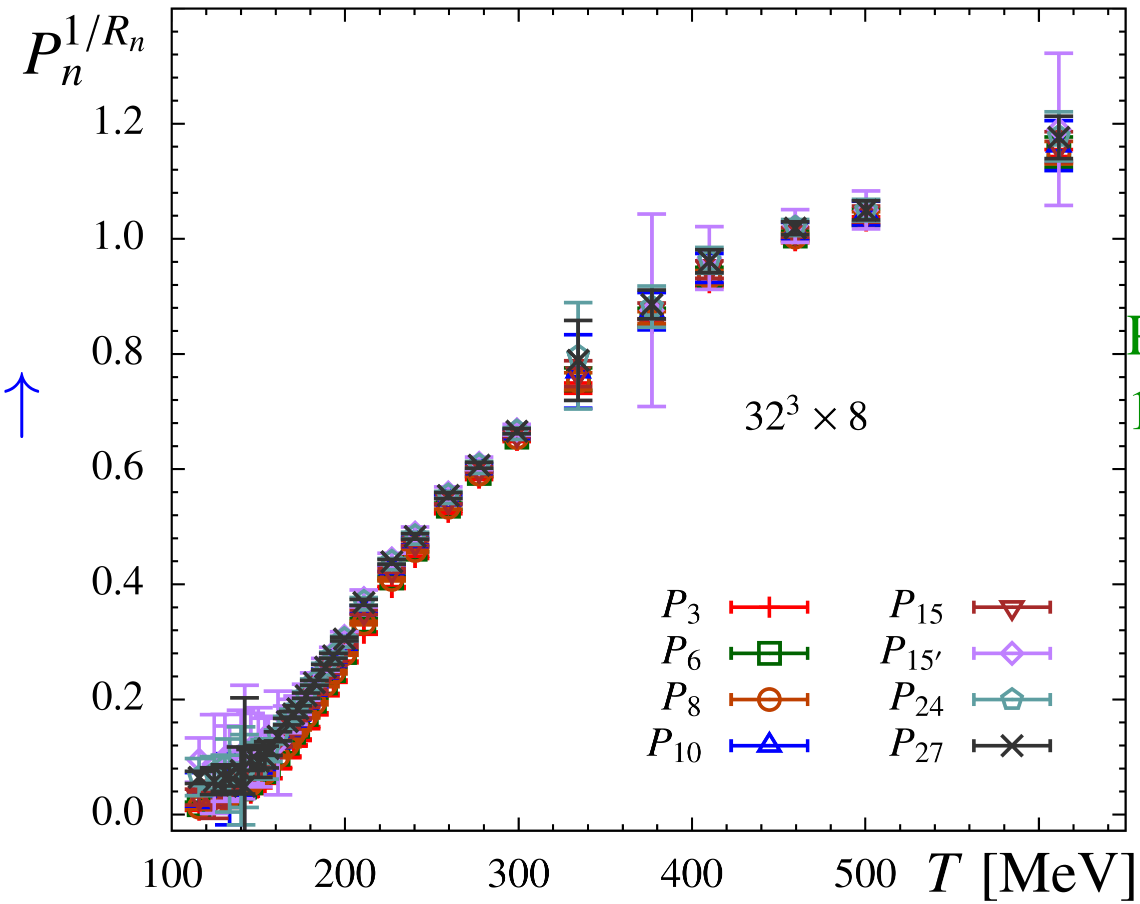
Technical aside

Can measure renormalized Polyakov loops in any representation.

Appear to satisfy universal scaling relation, both in pure glue & with quarks

Pure glue: Gupta, Hubner & Kaczmarek 0711.2251. With 2+1 flavors:

$(\langle \ell_{\mathcal{R}} \rangle)^{1/C_{\mathcal{R}}} \uparrow$

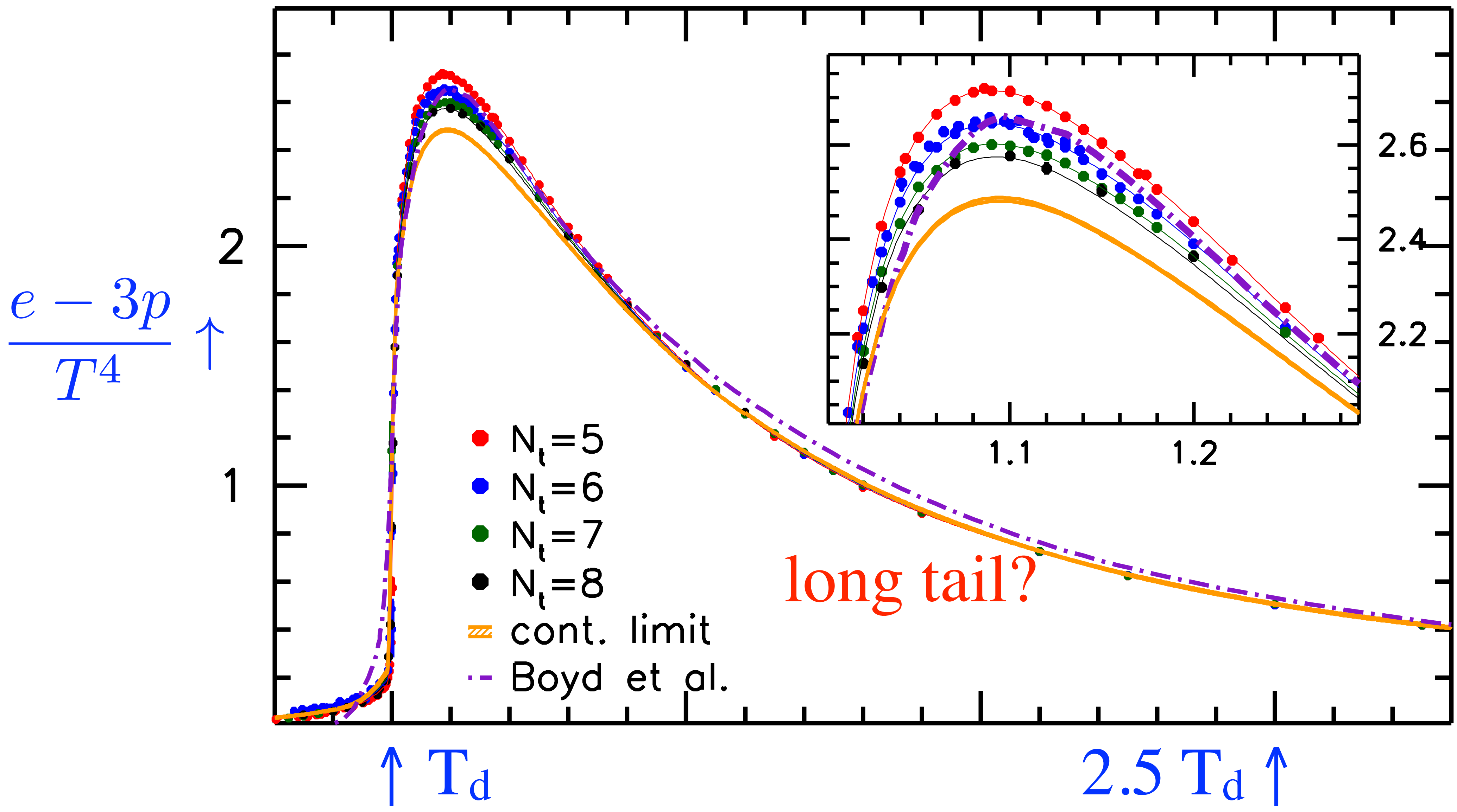


Petreczky & Schadler, 1509.07874

Lattice: first order

"Pure" SU(3), no quarks. Weakly first order. Peak in $(e-3p)/T^4$, just above T_d.

Borsanyi, Endrodi, Fodor, Katz, & Szabo, 1204.6184



Lattice: deconfined strings

$T_d \rightarrow 4\,T_d$: *leading* correction

to ideal gas, $\sim T^4$, is $\sim T^2$

not a bag constant, T^0

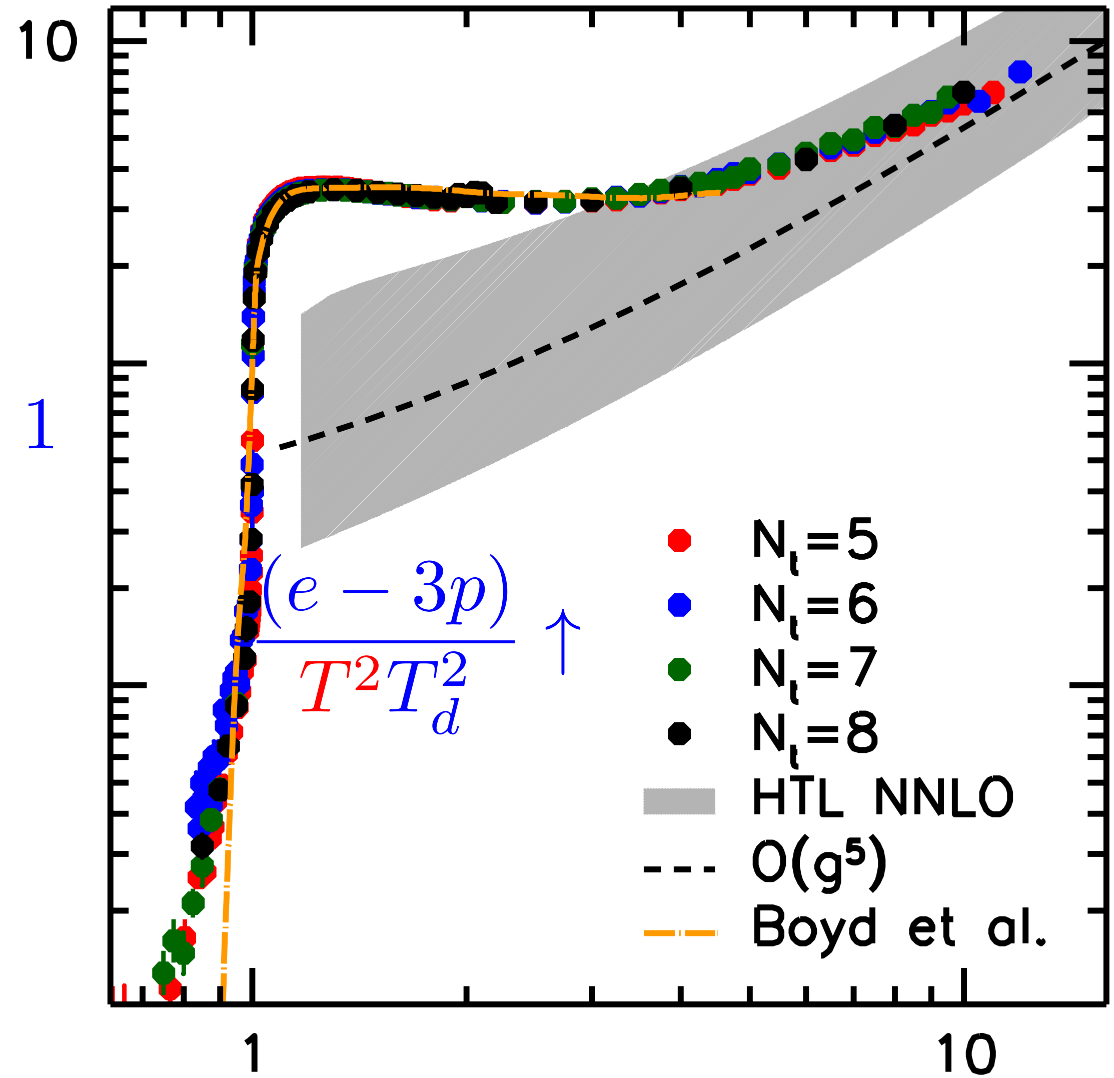
$$p(T) \sim \#(T^4 - c\,T^2\,T_d^2)\;,\; c \approx 1$$

Borsanyi, Endrodi, Fodor, Katz,
& Szabo, 1204.6184

Term $\sim T^2$ is like the

pressure of *deconfined* strings

$\sim$ constant for all SU(N)



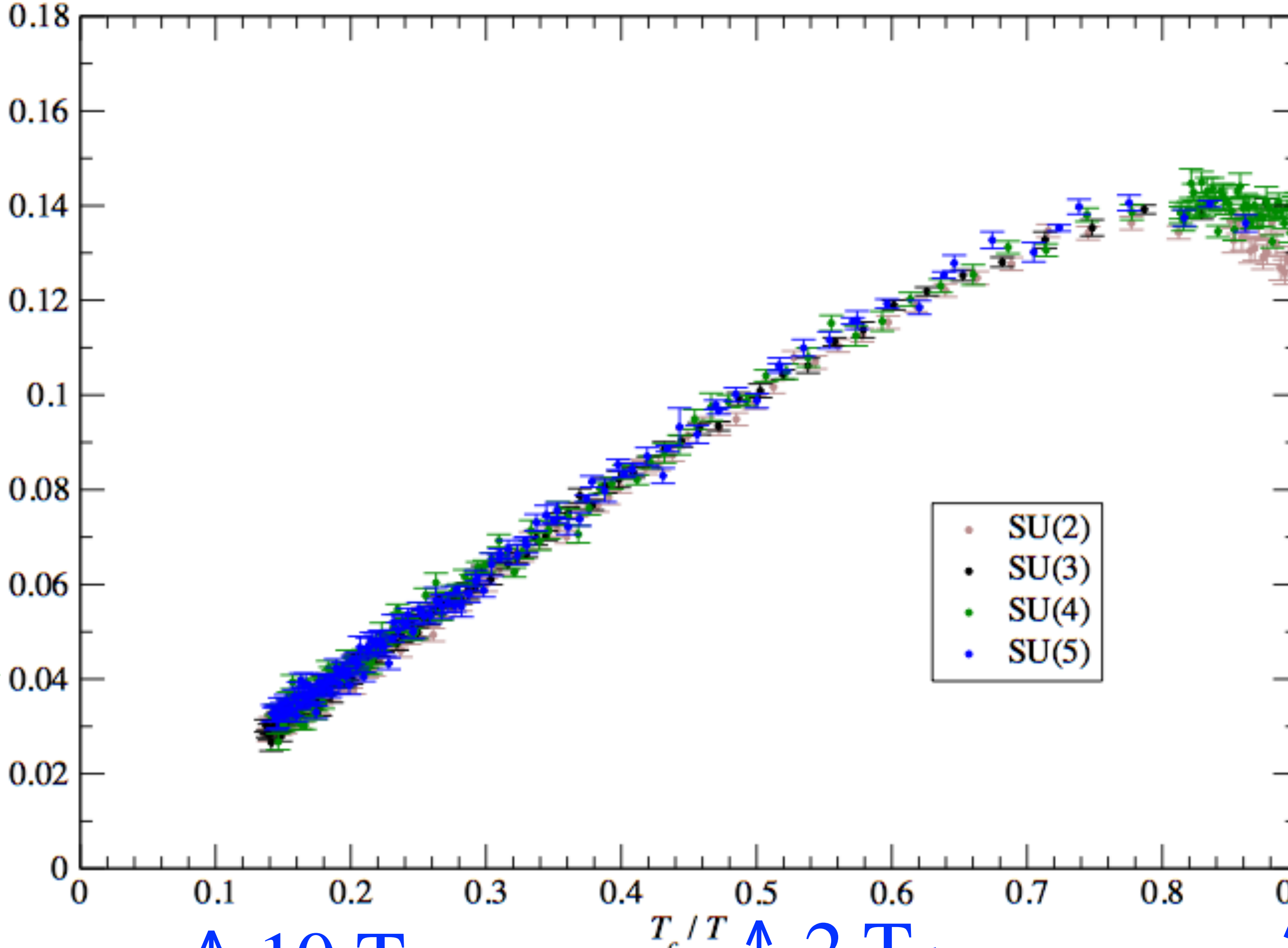
↑ T_d $10\,T_d$ ↑

Lattice: deconfined strings for SU(N), 2+1 dimensions

In 2+ 1 dimensions, hidden scaling again ~ T^2: *not* a mass term, ~ m^2 T:

$$p(T) \sim \#(T^3 - c\,T^2\,T_d)\ ,\ c \approx 1$$

$$\frac{1}{N^2-1}\,\frac{e-2p}{T^3} \uparrow$$



$T_d/T \rightarrow$

↑ 10 T_d ↑ 2 T_d ↑ 1.1 T_d

Caselle, Castagnini, Feo, Gliozzi, Gursoy, Panero, Schafer, 1111.0580

Matrix model for pure glue theories

Path to confinement

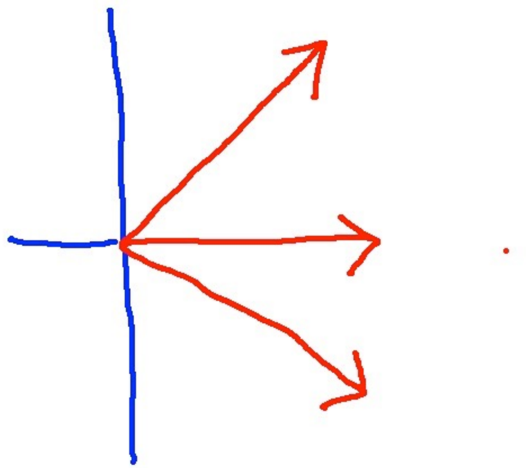
$\langle \textit{Wilson line} \rangle$ is a matrix, so diagonalize. SU(3): 2 diagonal generators, t_3 & t_8 :

$$t_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \;,\; t_8 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Above, paths along t_8, give Z(3) transf's. Now consider paths $\sim t_3$:

$$\mathbf{L} = \mathrm{e}^{2\pi i\, q\, t_3/3} \quad = \begin{pmatrix} \mathrm{e}^{2\pi i\, q/3} & 0 & 0 \\ 0 & \mathrm{e}^{-2\pi i q/3} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\ell = \frac{1}{3}\,\mathrm{tr}\,\mathbf{L} = \frac{1}{3}\left(1 + 2\cos\left(\frac{2\pi q}{3}\right)\right)$$



Confining vacuum: q = 1, $\langle \textit{loop} \rangle = 0$

Matrix model for pure glue

Perturbative potential is ideal gas + *previous potential for q*

$$\mathcal{V}_{pert}(q) = \frac{2\pi^2}{3} T^4 \left(-\frac{4}{15} + \sum_{a,b} q_{ab}^2 (1 - q_{ab})^2\right) \;,\; q_{ab} = |q_a - q_b|_{\mathrm{mod}\,1}$$

Assume non-pert. potential ~ T^2:

$$\mathcal{V}_{non}(q) = \frac{2\pi^2}{3} T^2\, T_d^2 \sum_{a,b} \left(-\, c_1 q_{ab}(1 - q_{ab}) - c_2 q_{ab}^2 (1 - q_{ab})^2 + \frac{4}{15}\, c_3\right)$$

From lattice data, constant term ~ c_3 most important for $T > 1.2\ T_d$.

Thus expect that the q's only matter for $T < 1.2\ T_d$: *narrow* transition region

Also added a bag constant B (helps with latent heat, not essential)

Dumitru, Guo, Hidaka, Korthals-Altes & RP, 1011.3820 & 1205.0137 + ….

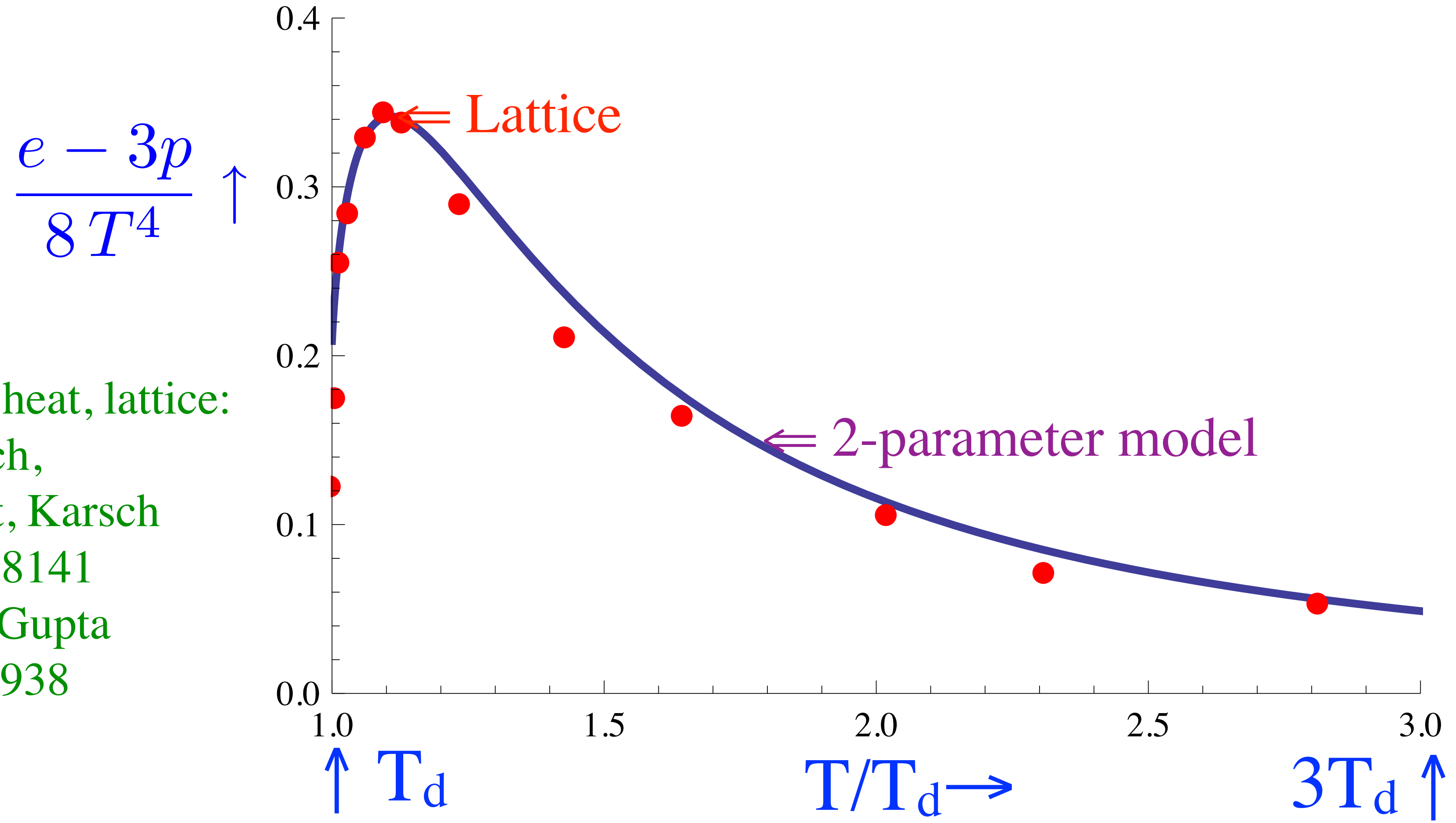
Matrix model: parameters from the lattice

Choose 2 free parameters to fit:
latent heat at T_c, $(e-3p)/T^4$ at large T

$c_1 = .88$, $c_2 = .55$, $c_3 = .95$

Reasonable value for bag constant B:

$T_d = 270$ MeV, $B \sim (262 \text{ MeV})^4$

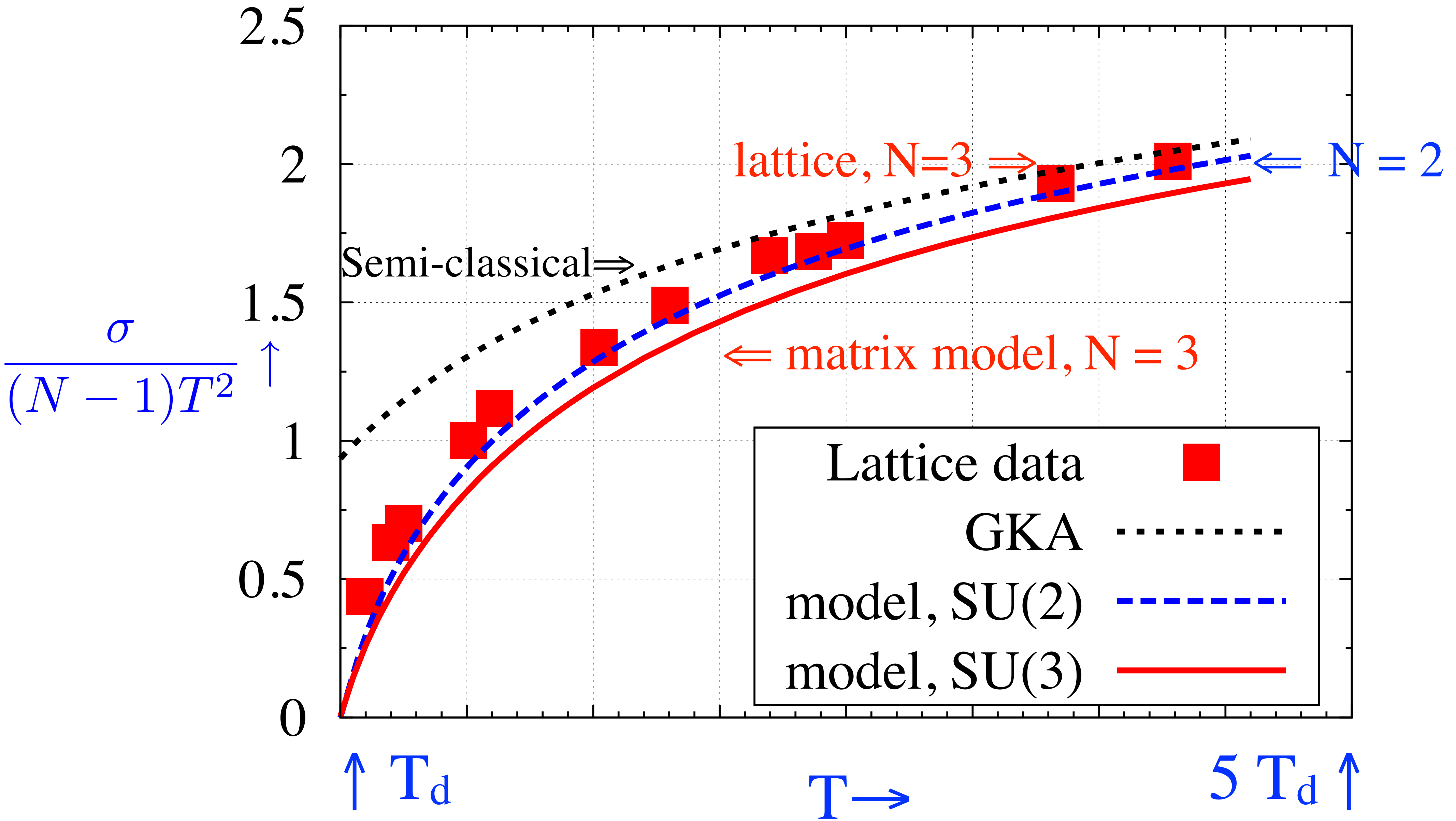


Latent heat, lattice:
Beinlich,
Peikert, Karsch
lat/9608141
Datta, Gupta
1006.0938

Matrix model: interface tension vs lattice

Matrix model works well:

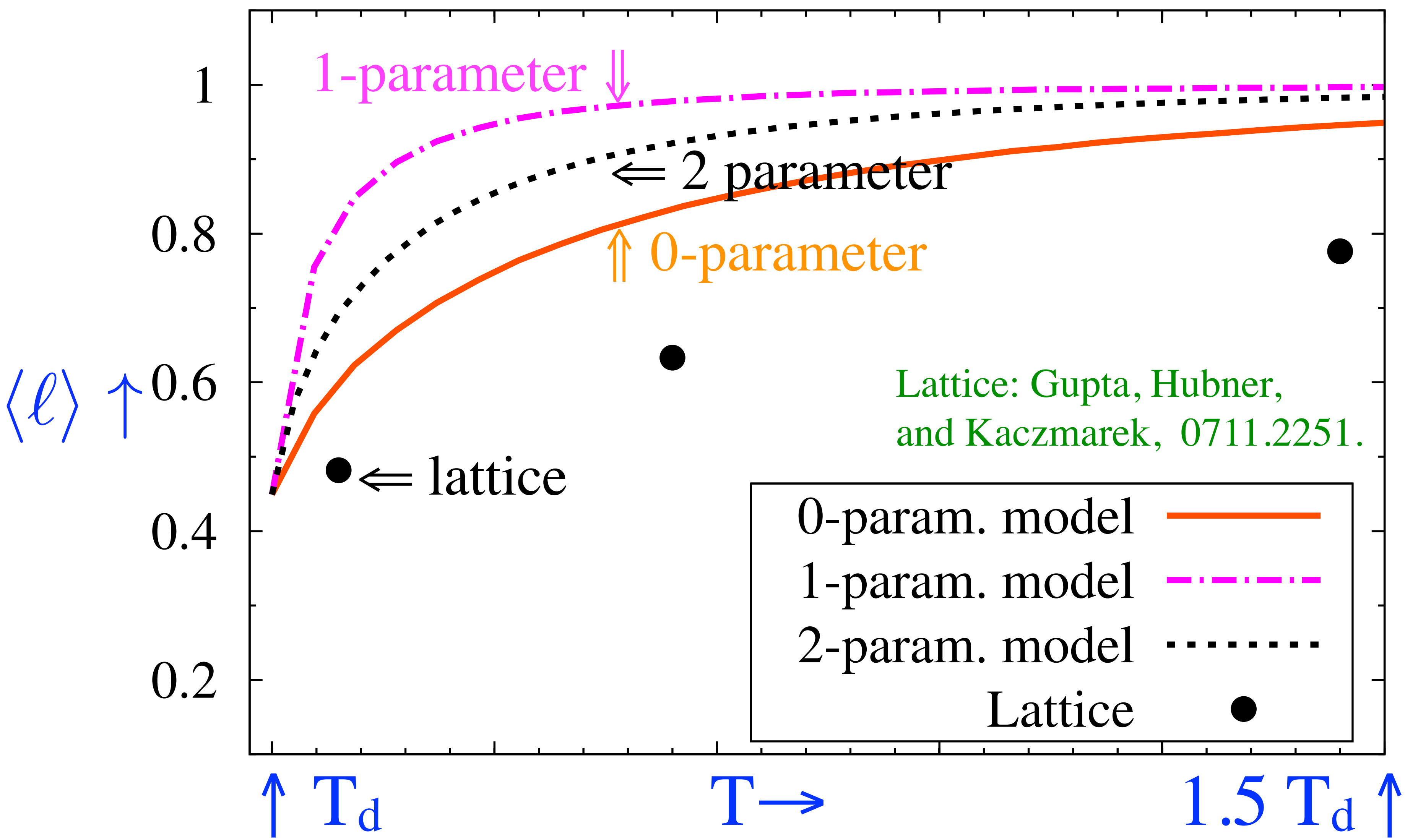
Lattice: de Forcrand, D'Elia, & Pepe, lat/0007034; de Forcrand & Noth lat/0506005



Matrix model: Polyakov loop vs lattice

Renormalized Polyakov loop from lattice *nothing* like Matrix Model

Model: transition region *narrow*, to ~ 1.2 T_d. Lattice: loop *wide*, to ~ 4.0 T_d.



Birdtrack diagrams for SU(N)

Another basis…

Above t_3 picks out a given direction in color space….why?

Need a new basis with no preferred direction. SU(2): three gen.'s. Two ladder:

$$t^{12} = \sigma^+ = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \; , \; t^{21} = \sigma^- = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

and one diagonal. Be perverse, and add *two*. N.B.: $t^{11} + t^{22} = 0$

$$t^{11} \sim \sigma^3 = \frac{1}{2\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \; , \; t^{22} \sim -\sigma^3 = \frac{1}{2\sqrt{2}} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

Perverse, but no preferred direction. Normalization weird with *one* extra gen.:

$$\mathrm{tr}\, t^{12} t^{21} = \frac{1}{2} \; , \; \mathrm{tr}(t^{12})^2 = \mathrm{tr}(t^{21})^2 = 0$$

$$\mathrm{tr}\, (t^{11})^2 = \mathrm{tr}(t^{22})^2 = \frac{1}{4} \; , \; \mathrm{tr}(t^{11} t^{22}) = -\frac{1}{4}$$

Weird basis

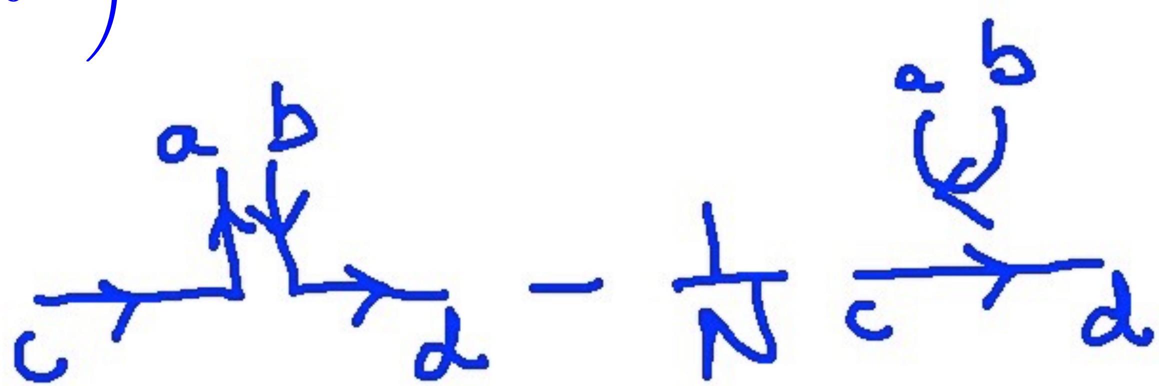
For SU(3), take a basis with *one* extra diagonal generator ($t^{11} + t^{22} + t^{33} = 0$)

$$t^{33} = t_8 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} \; , \; t^{22} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \; , \; t^{11} = \begin{pmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

No gen. like SU(2), diag(1,-1,0):
no preferred direction. Overcomplete by *one* generator.

Diagramatically, easy to generalize to arbitrary SU(N):

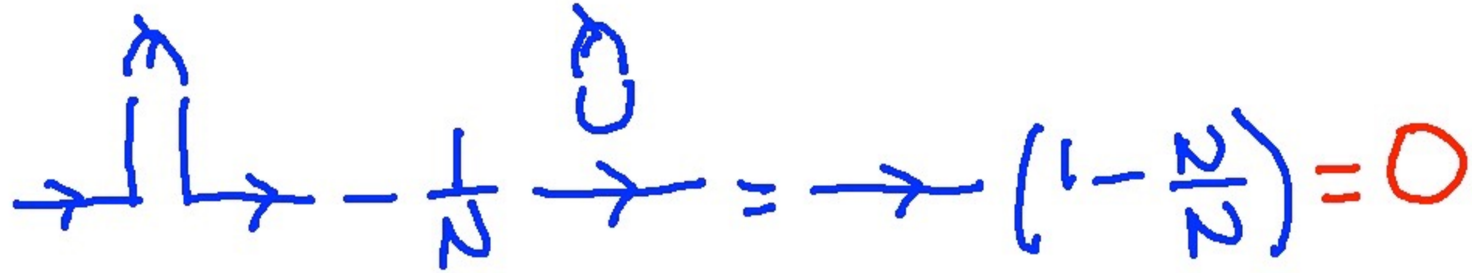
$$(t^{ab})_{cd} = \frac{1}{\sqrt{2}} \left(\delta^{ac}\, \delta^{bd} - \frac{1}{N}\, \delta^{ab}\, \delta^{cd} \right)$$



Birdtracks = double line basis

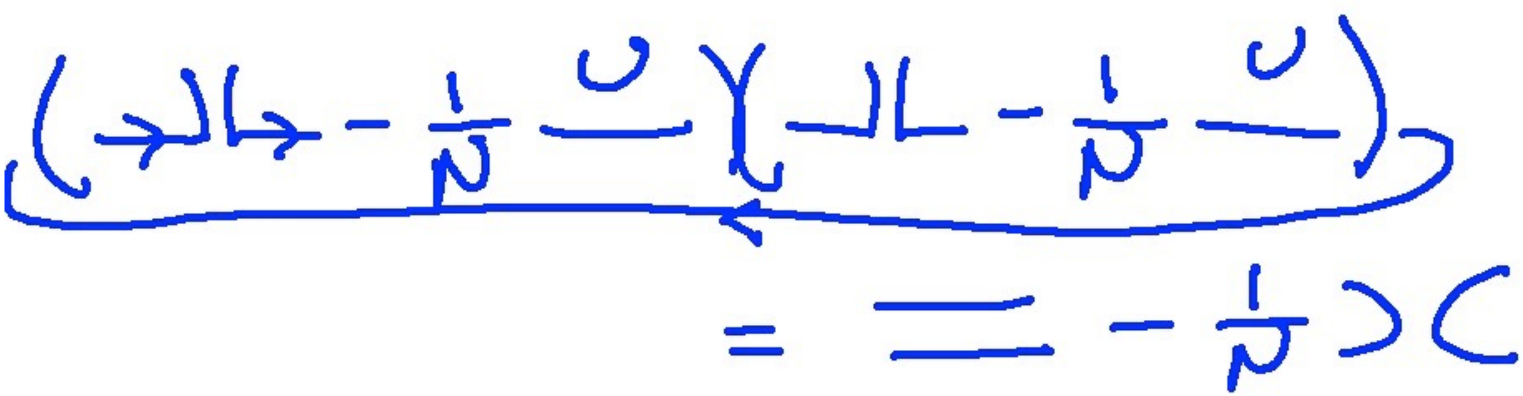
Basis overcomplete by one:

$$\sum_{a=1}^{N} t^{aa} = \sum_{a,b} t^{ab}\delta^{ba} =$$



Product of two generators is a projector:

$$\mathrm{tr}(t^{ab}t^{cd}) = \frac{1}{\sqrt{2}}(t^{ab})_{dc}$$

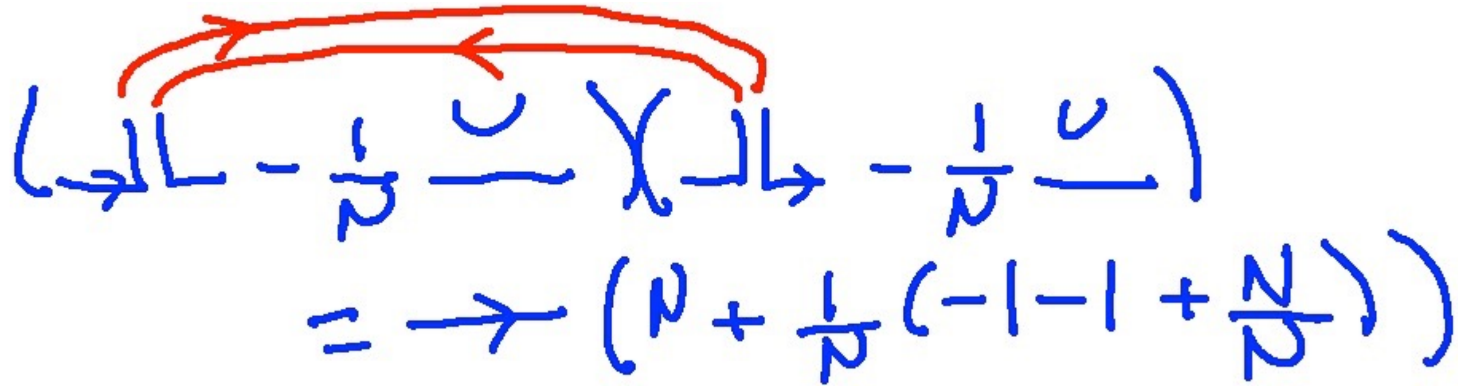


P. Cvitanovic, http://birdtracks.eu ; Y. Hidaka & RDP 0803.0453

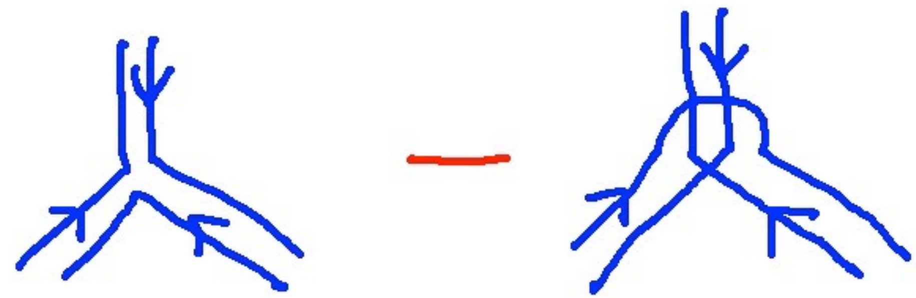
And on with birdtracks

Can derive *arbitrary* SU(N) identities by pecking:

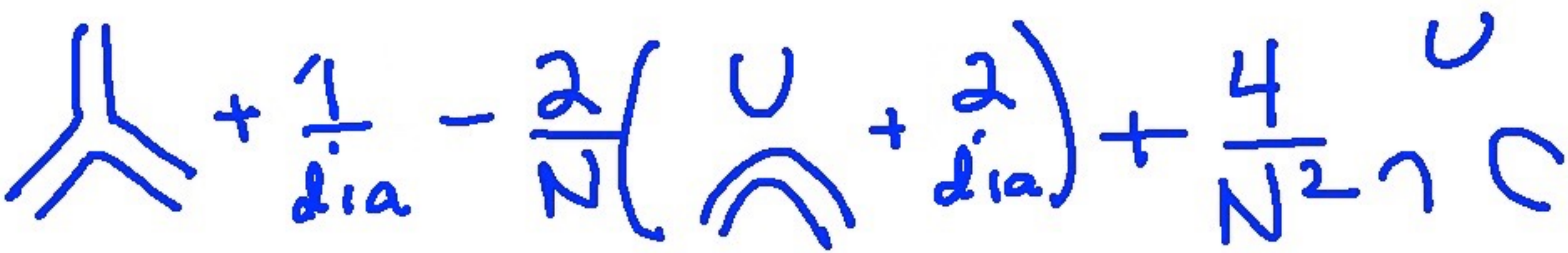
$$(t^{ab}t^{ba})_{cd} = \frac{N^2 - 1}{2N}\delta_{cd}$$



Antisymmetric $f^{ab,cd,ef}$ is simple:

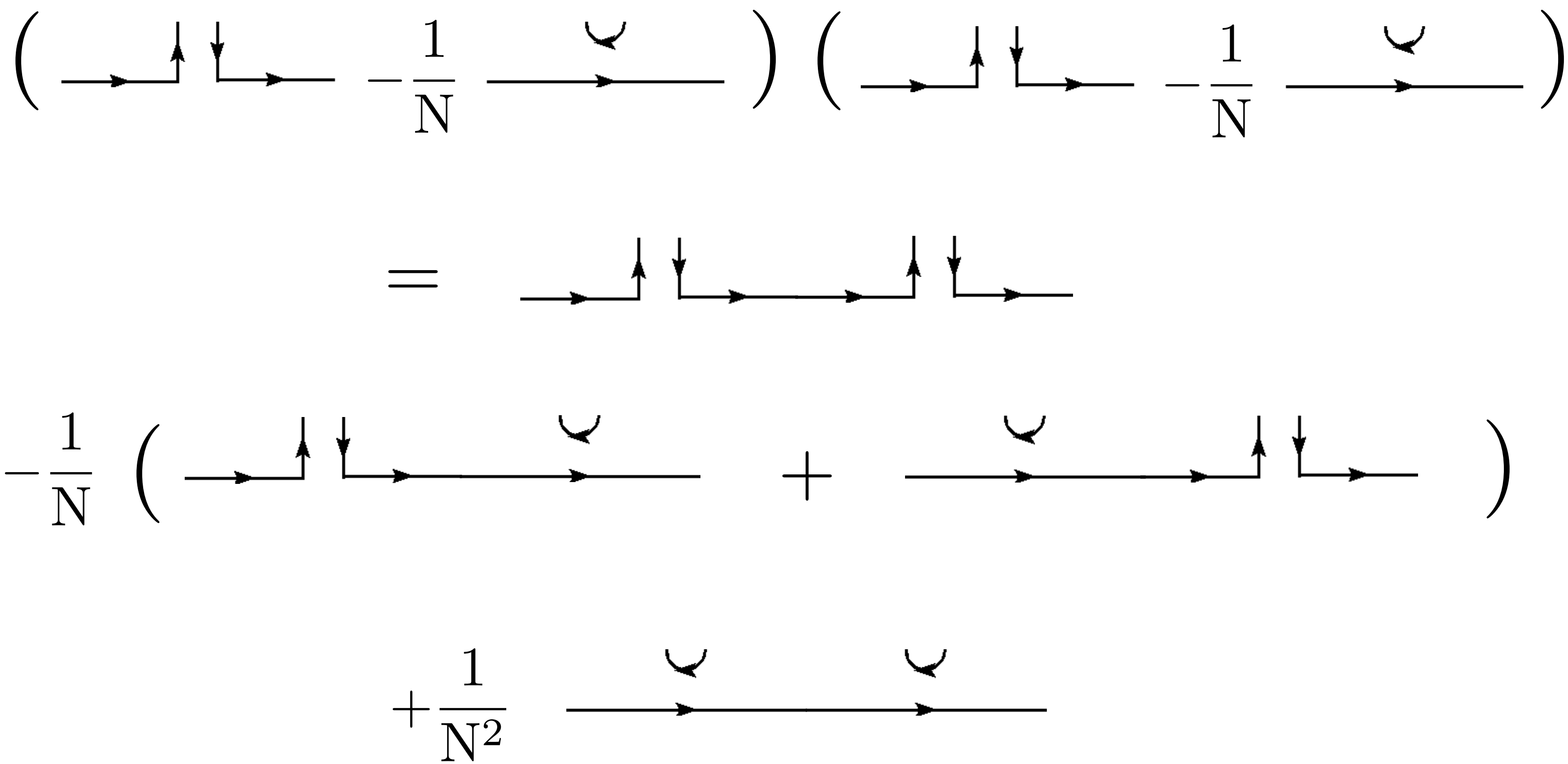


Symmetric $f^{ab,cd,ef}$ is not, because of the traces: *this* is why SU(N) is hard!



Group identities from birdtracks

$$(t^{ab})_{eg}\,(t^{cd})_{gf} =$$



Just by drawing arrows, can show the standard relation:

$$\sum_{a,b=1}^{N} (t^{ab}\ t^{ba})_{cd} = \frac{N^2-1}{N}\,\delta_{cd}$$

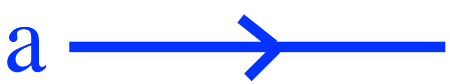
Birdtracks = double line

At large N, *trival*: drop all trace terms! Double line notation of 't Hooft.

Consider expanding about some background field:

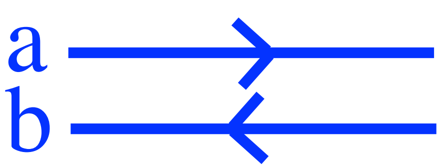
$$\left(A_0^{cl}\right)_{ab} = \frac{2\pi T}{g}\, q_a\, \delta^{ab}$$

At any N, quark propagator has a single line, with one color index



$$i\, D_0^{cl} = 2\pi T(n + 1/2 + q_a)$$

At large N, gluons have two lines, with two color indices



$$i\, D_0^{cl} = 2\pi T(n + q_a - q_b)$$

Pure gauge transition for SU(N) at large N:

Gross-Witten-Wadia?

QCD on a femtosphere

Consider pure SU(∞) on a spatial sphere so small that coupling is small

Sundberg, th/9908001;

Aharony, Marsano, Minwalla, Papadodimas, Van Raamsdonk, th/0310285; th/0508077

Dumitru, Lenaghan, RDP, ph/0410294

Integrate out modes with $J \neq 0$, obtain eff. theory for static modes, matrix model

Consider eigenvalues of Wilson line, $\mathbf{L} = \exp(2\,\pi\, i\, \mathbf{q})$

Take $A^i_0 \sim q^i$, $i = 1 \ldots N$. discrete sum $\Sigma_i \Rightarrow \int dq\, \varrho(q)$.

$$\# \left| \int dq\, \rho(q)\, e^{2\pi i\, q} \right|^2 + \int dq \int dq'\, \rho(q)\, \rho(q') \log \left| e^{2\pi i q} - e^{2\pi i q'} \right|$$

Solve by usual large N tricks. At T_d, eigenvalue density is

$$\rho(q) = 1 + \cos(2\pi\, q) \quad , \quad q : -1/2 \to 1/2$$

N.B. in 2-dim.'s, Gross, Witten, & Wadia found 3rd order transition in lattice β.

Here, at any temperature, find 3rd order transition when

$$\ell = \frac{1}{N}\,\mathrm{tr}\,\mathbf{L} = \frac{1}{2}$$

Gross-Witten-Wadia transition at N=∞

Solution at N=∞: "critical first order" transition - both first *and* second order

Latent heat *non*zero ~ N^2. *And* specific heat diverges, $C_v \sim 1/(T-T_c)^{3/5}$

Potential function of *all* tr $\mathbf{L}^n$, n = 1, 2.... But at T_d^+, only *first* loop is nonzero:

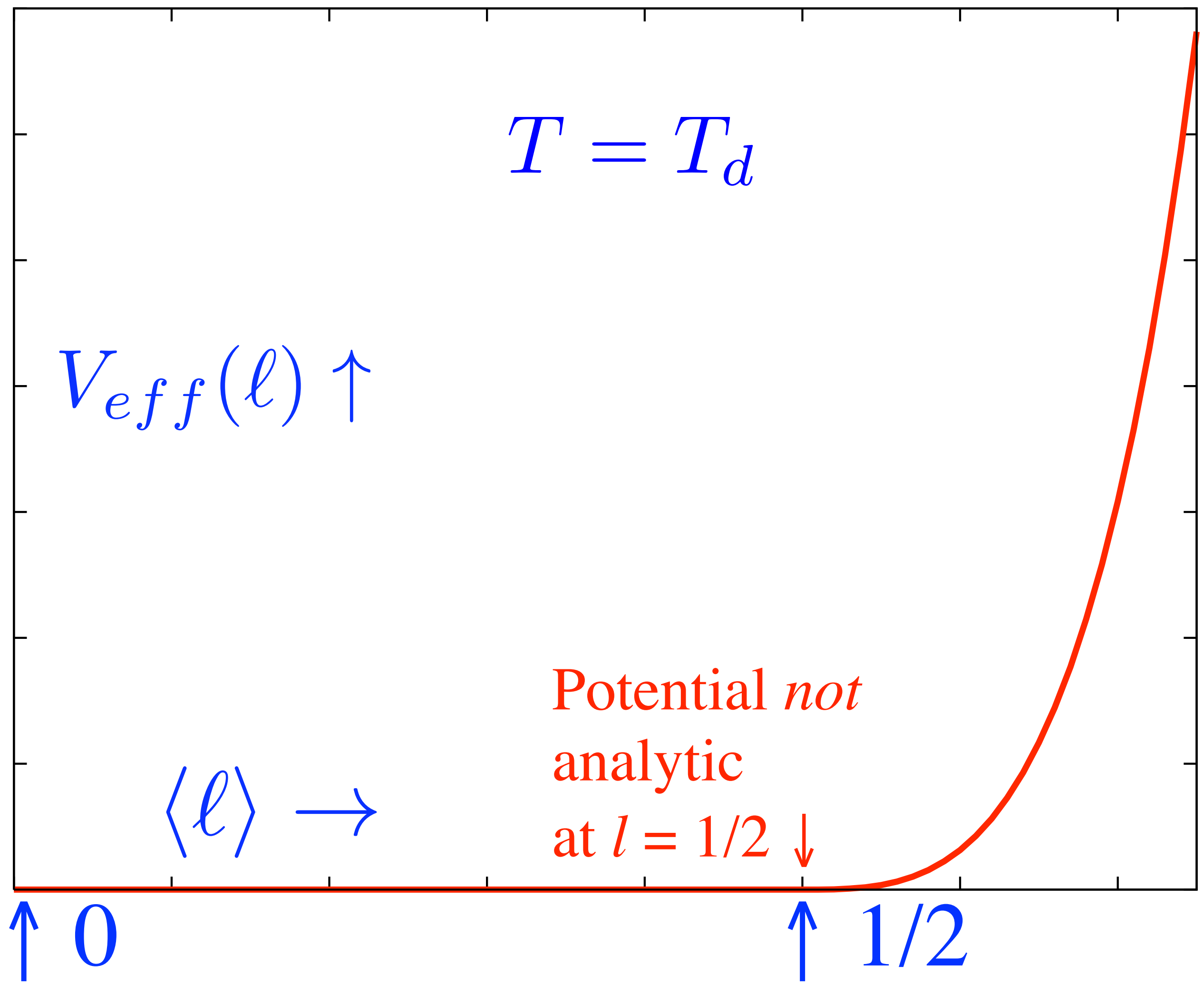
$$\ell = \frac{1}{N}\,\mathrm{tr}\,\mathbf{L}$$

$$\ell(T_c^-) = 0$$

$$\ell(T_c^+) = \frac{1}{2}$$

But V_{eff} *flat* between them!

tr $\mathbf{L}^n$ (T_d) = 0 , n ≥ 2



Above *only* for g=0: to ~ g^4, standard 1st order transition. So GWW curiosity?

General matrix model, 3 colors

Remember model for three colors

Meisinger, Miller, & Ogilvie, ph/0108009.

A. Dumitru, Y. Guo, Y. Hidaka, C. Korthals-Altes & RDP, 1011.3820, 1205.0137;

K. Kashiwa, V. Skokov & RDP, 1205.0545; K. Kashiwa & RDP, 1301.5344.

Simple ansatz: constant, diagonal A_0:

$$A_0^{ij} = \frac{2\pi T}{g}\, q_i\, \delta^{ij}\ ,\ i, j = 1 \ldots N$$

At 1-loop order, perturbative potential

$$V_{pert}(q) = \frac{2\pi^2}{3}\, T^4 \left(-\frac{4}{15}(N^2-1) + \sum_{i,j} q_{ij}^2 (1-q_{ij})^2 \right)\ ,\ q_{ij} = |q_i - q_j|$$

Non-perturbative potential ~ $T^2\, T_d^2$:

$$V_{non}(q) = \frac{2\pi^2}{3}\, T^2 T_d^2 \left(-\frac{c_1}{5} \sum_{i,j} q_{ij}(1-q_{ij}) - c_2 \sum_{i,j} q_{ij}^2 (1-q_{ij})^2 + \frac{4}{15}\, c_3 \right) + B T_d^4$$

Matrix models at infinite N

Solve SU(N) at N=∞: RDP & V. Skokov, 1206.1329; Nishimura, RDP, & Skokov, to appear
Interface tensions: S. Lin, RDP, & V. Skokov, 1301.7432

$$V_{\text{eff}}(q) = d_1\, V_1 + d_2\, V_2$$

$$V_n(q) = \int dq \int dq'\ \rho(q)\ \rho(q')\ |q - q'|^n (1 - |q - q'|)^n$$

Take derivatives of equation of motion, at T_d solution

$$\rho(q) = 1 + \ \cos(2\pi\, q) \ \ , \ \ q : -1/2 \to 1/2$$

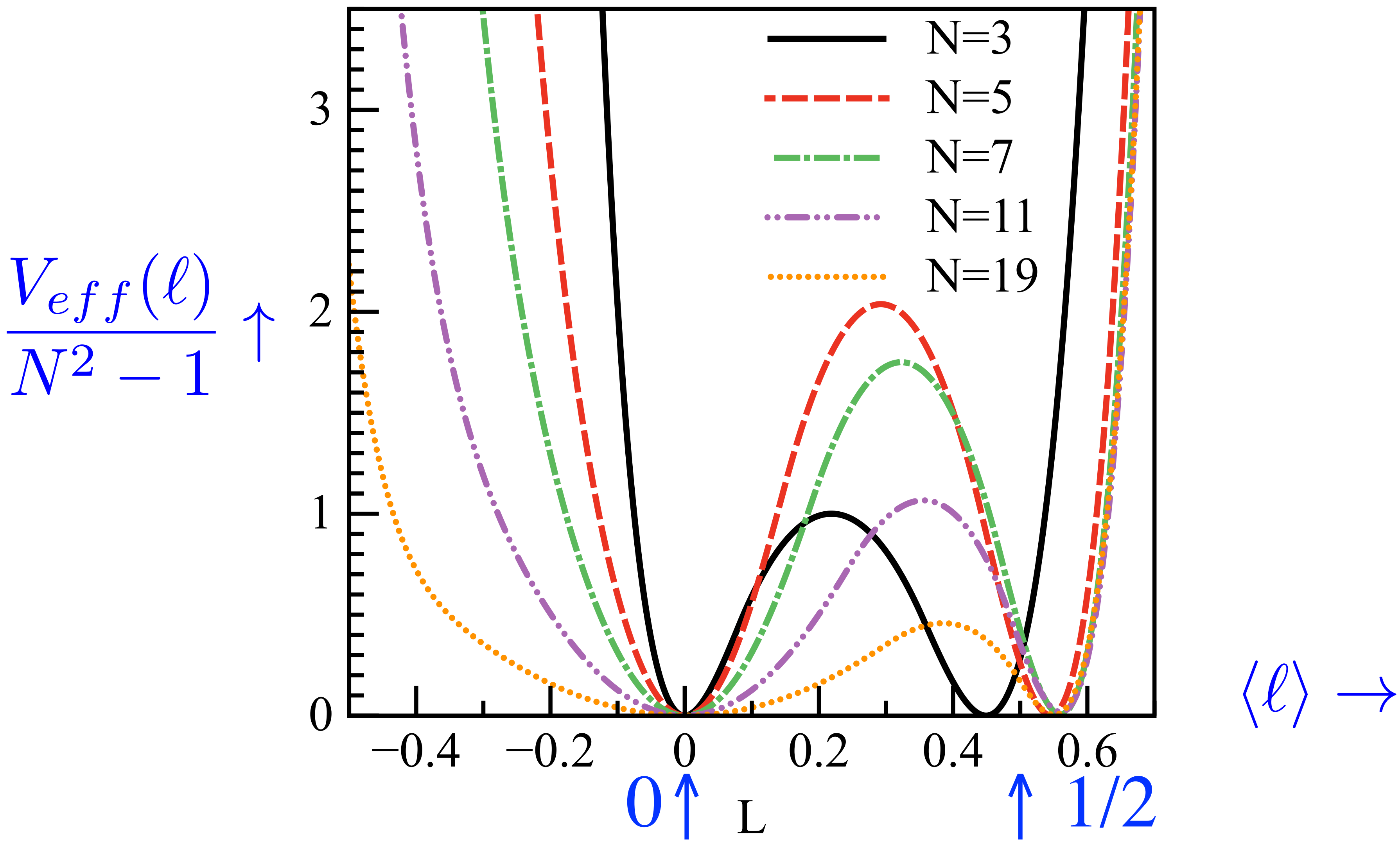
At T_d, solution *identical* to GWW model on a femtosphere!

Solution differs away from T_d. *But why same solution at T_d?* V_{eff} *very* different.

Is Gross-Witten-Wadia an infrared stable fixed point for pure gauge SU(∞)?

Remnants of Gross-Witten-Wadia at finite N?

At finite N, solve model numerically. Find two minima, at 0 and ~ 1/2.
Standard first order transition, with barrier & interface tension *non*zero
Barrier disappears at infinite N: so interface tensions *vanish* at infinite N
Below: potential /(N^2-1), versus tr **L** .



Signs of GWW at finite N: interface tensions *small* at T_d?

Consider maximum of previous figure, versus number of colors:
increases by ~ 2 from N = 3 to 5, then *decreases* monotonically as N increases
Perhaps: non-monotonic behavior of order-disorder interface tension with N?
Below: maximum in potential $/(N^2-1)$, versus tr $\mathbf{L}$.

Lattice: order-disorder interface tension α^{od} at T_d:
Lucini, Teper, Wegner, lat/0502003

$$\frac{\alpha^{od}}{N^2 T_d^3} = .014 - \frac{.10}{N^2}$$

Coefficients *small*, χ^2 *large*, ~ 2.8.
Non-monotonic behavior of α^{od}/N^2?
't Hooft loops also *small* near T_d

Remnants of Gross-Witten-Wadia fixed point at finite N?